Westpac

AUSTRALIAN MATHEMATICS COMPETITION
FOR THE WESTPAC AWARDS

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

WEDNESDAY 25 JULY 2007

SENIOR DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 11 AND 12
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Senior Division

Questions 1 to 10, 3 marks each

1. $2(5.61 - 4.5)$ equals

(A) 3.1 (B) 10.48 (C) 2 (D) 2.22 (E) 6.72

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2. If $2^n + 2^n = 2^m$, then

(A) $n + n = m$ (B) $n + 1 = m$ (C) $4n = m$ (D) $m + 1 = n$ (E) $n^2 = m$

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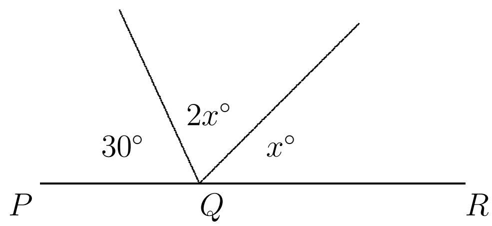
3. PQR is a straight line. The value of x is

(A) 30 (B) 45 (C) 50 (D) 60 (E) 150



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4. Of the following, which is the largest fraction?

(A) $\frac{7}{15}$ (B) $\frac{3}{7}$ (C) $\frac{6}{11}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

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5. Nicky started a mobile phone call at 10:57 am. The charge for the call was 89 cents per minute and the total cost for the call was \$6.23. Nicky's call ended at

(A) 11:27 am (B) 11:14 am (C) 11:04 am (D) 11:46 am (E) 11:05 am

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6. The straight lines with equations $2x + y = q$ and $y = x - p$ meet at the point $(2, k)$. The value of $p + q$ is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

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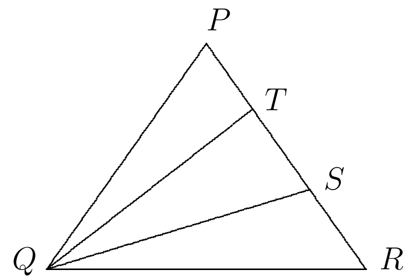
7. PQR is an equilateral triangle, QS and QT divide $\angle PQR$ into three equal parts. The size of $\angle QTS$, in degrees, is

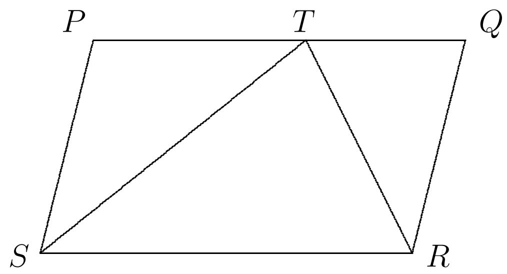


(A) 40 (B) 70 (C) 80 (D) 90 (E) 100

8. Jane's age is a prime number. Andy's age has 8 factors and he is one year older than Jane. Of the following numbers, which could be the sum of their ages?

(A) 27 (B) 39 (C) 75 (D) 87 (E) 107

9. $PQRS$ is a parallelogram and T lies on PQ such that $PT : TQ = 3 : 2$. The ratio of the area of $PTRS$ to the area of $PQRS$ is



(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 4 : 5 (E) 5 : 6

10. Five positive integers have a mean of 5, a median of 5 and just one mode of 8. What is the difference between the largest and the smallest integers in the set?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Questions 11 to 20, 4 marks each

11. Dad filled his sprayer with 8 litres of water. He then added 16 drops of insecticide instead of the recommended dosage of 32 drops. After using 2 litres of the spray, he realised his mistake, refilled the sprayer with another 2 litres of water and added a sufficient number of drops of insecticide to reach the recommended concentration.The number of extra drops that dad needed to add was

(A) 20 (B) 12 (C) 8 (D) 16 (E) 24

12. The game of *Four Tofu* is played on a 4×4 grid. When completed, each of the numbers 1, 2, 3 and 4 occurs in each row and column of the 4×4 grid and also in each 2×2 corner of the grid.
When the grid shown is completed, the sum of the four numbers in the corners of the 4×4 grid is

	2		
			1
	1	3	
4			

(A) 13 (B) 11 (C) 15 (D) 12 (E) 10

13. Holly writes down all the two-digit numbers which can be formed using the digits 1, 3, 7 and 9 (including 11, 33, 77 and 99). Warren selects one of these numbers at random. What is the probability that it is prime?

(A) $\frac{5}{8}$ (B) $\frac{1}{2}$ (C) $\frac{9}{16}$ (D) $\frac{11}{16}$ (E) $\frac{3}{4}$

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14. Two rectangular garden beds have a combined area of $40\,\text{m}^2$. The larger bed has twice the perimeter of the smaller and the larger side of the smaller bed is equal to the smaller side of the larger bed. If the two beds are not similar, and if all edges are a whole number of metres, what is the length, in metres, of the longer side of the larger bed?

(A) 7 (B) 8 (C) 10 (D) 14 (E) 27

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15. I take a two-digit positive number and add to it the number obtained by reversing the digits. For how many two-digit numbers is the result of this process a perfect square?

(A) 1 (B) 3 (C) 5 (D) 8 (E) 10

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16. Ann, Bill and Carol sit on a row of 6 seats. If no two of them sit next to each other, in how many different ways can they be seated?

(A) 12 (B) 24 (C) 18 (D) 36 (E) 48

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17. The number of integer solutions of the equation

$$(x^2 - 3x + 1)^{x+1} = 1$$

is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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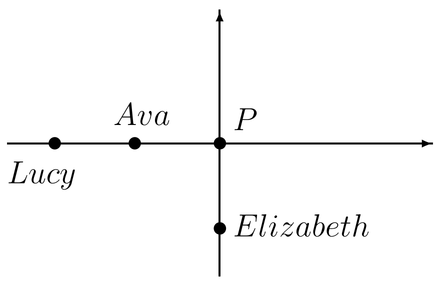
18. Ava and Lucy both jog at 8 km/h along a straight path with Lucy staying 12 m behind Ava. Elizabeth jogs at 6 km/h along a straight path which meets the first path at right-angles at P. When Elizabeth is at P she is the same distance from Ava as from Lucy.



When Ava was first 50 m from P, how far, in metres, was Elizabeth from P?

(A) 40 (B) 42 (C) 44 (D) 46 (E) 48

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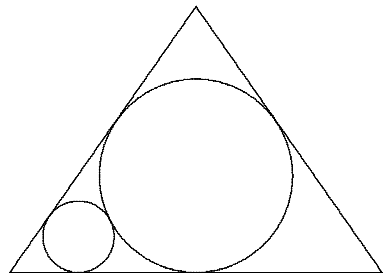
19. On a 3×5 chessboard, a counter can move one square at a time along a row or a column, but not along any diagonal. Starting from some squares, it can visit each of the other 14 squares exactly once, without returning to its starting square. Of the 15 squares, how many could be the counter's starting square?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

20. The inscribed circle of an equilateral triangle has radius 1 unit. A smaller circle is tangent to this circle and to two sides of the triangle as shown. The radius of this smaller circle is



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{6}$ (D) $\frac{\sqrt{3}-1}{2}$ (E) $\frac{1}{5}$

Questions 21 to 25, 5 marks each

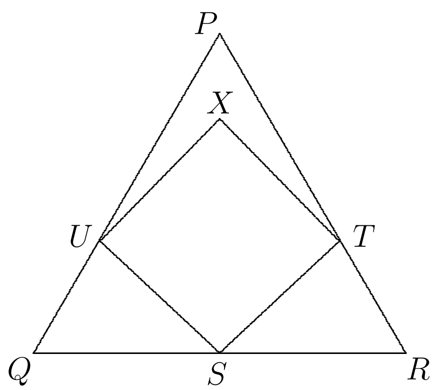
21. There are four lifts in a building. Each makes three stops, which do not have to be on consecutive floors or include the ground floor. For any two floors, there is at least one lift which stops on both of them. What is the maximum number of floors that this building can have?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 12

22. A bee can fly or walk only in a straight line between any two corners on the inside of a cubic box of edge length 1. The bee managed to move so that it visited every corner of the box without passing through the same point twice in the air or on the wall of the box. The largest possible length of such a path is

(A) $2 + 5\sqrt{2}$ (B) $1 + 6\sqrt{2}$ (C) $7\sqrt{2}$ (D) $\sqrt{3} + 6\sqrt{2}$ (E) $4\sqrt{3} + 3\sqrt{2}$

23. PQR is an equilateral triangle with side length 2. S is the midpoint of QR and T and U are points on PR and PQ respectively such that $STXU$ is a square. The area of this square is



(A) $6 - 3\sqrt{3}$ (B) $\frac{5 - 2\sqrt{3}}{2}$ (C) $\frac{3}{4}$ (D) $\frac{2\sqrt{2}}{3}$ (E) $\frac{1 + \sqrt{2}}{2}$

24. How many functions $f(x) = ax^2 + bx + c$ are there with the property that, for all x, $f(x) \times f(-x) = f(x^2)$?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

25. Let $(\sqrt{2}+1)^{2007} = a + b\sqrt{2}$, where a and b are integers. The highest common factor of b and 81 is

(A) 1 (B) 3 (C) 9 (D) 27 (E) 81

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. A rectangular area measuring 3 units by 6 units on a wall is to be covered with 9 tiles each measuring 1 unit by 2 units. In how many ways can this be done?

27. There are 42 points $P_1, P_2, P_3, \ldots, P_{42}$, placed in order on a straight line so that each distance from P_i to P_{i+1} is $\frac{1}{i}$ where $1 \le i \le 41$. What is the sum of the distances between every pair of these points?

28. A *lucky number* is a positive integer which is 19 times the sum of its digits. How many different lucky numbers are there?

29. On my calculator screen the number 2659 can be read upside down as 6592. The digits that can be read upside down are 0, 1, 2, 5, 6, 8, 9 and are read as 0, 1, 2, 5, 9, 8, 6 respectively. Starting with 1, the fifth number that can be read upside down is 8 and the fifteenth is 21. What are the last three digits of the 2007th number that can be read upside down?

30. Consider the solutions (x, y, z, u) of the system of equations

$$\begin{aligned} x + y &= 3(z + u) \\ x + z &= 4(y + u) \\ x + u &= 5(y + z) \end{aligned}$$

where x, y, z and u are positive integers. What is the smallest value that x can have?

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