AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

THURSDAY 5 AUGUST 2010

JUNIOR DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 7 AND 8

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the **Answer Sheet** carefully. Ensure your name, school name and school year are filled in. It is your responsibility that the Answer Sheet is correctly coded.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the Answer Sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your Answer Sheet will be read by a machine. The machine will see all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the Answer Sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMC reserves the right to re-examine students before deciding whether to grant official status to their score.

Junior Division

Questions 1 to 10, 3 marks each

1. The value of $27 + 48 - 37$ is

(A) 32 (B) 38 (C) 48 (D) 52 (E) 68

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2. The value of $2^2 + 3^3$ is

(A) 31 (B) 10 (C) 11 (D) 25 (E) 17

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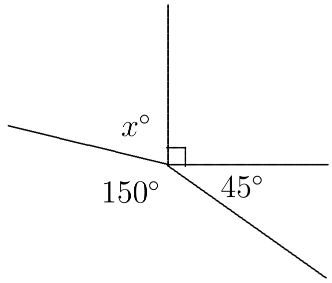
3. In the diagram, the value of x is

(A) 15 (B) 40 (C) 55 (D) 75 (E) 80



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4. A 55-minute school assembly ends at 10:05 am. At what time did it start?

(A) 9:15 am (B) 9:20 am (C) 9:10 am (D) 9:50 am (E) 10:50 am

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5. The value of $2010 - 20.10$ is

(A) 1990.09 (B) 1990.9 (C) 1989.09 (D) 1989.9 (E) 1998.9

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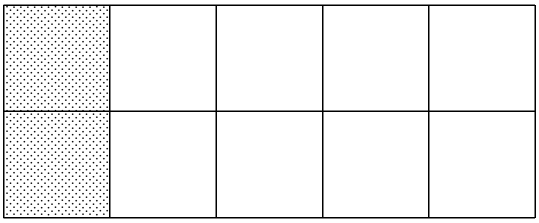
6. Which of the following is equal to $4 + \frac{1}{6} - \frac{2}{3}$?

(A) $3\frac{5}{6}$ (B) $3\frac{2}{3}$ (C) $4\frac{1}{3}$ (D) $3\frac{8}{9}$ (E) $3\frac{1}{2}$

7. The grey shaded tiles represent $\frac{1}{5}$ of the large rectangle. How many white tiles must be removed so that the grey tiles represent $\frac{1}{3}$ of the remaining shape?



(A) 2 (B) 3 (C) 4 (D) 6 (E) 7

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8. A bus is timetabled to stop outside my house at equal intervals throughout the day. It is now 3:25 pm and the last bus arrived 6 minutes ago, but it was 2 minutes late. The next bus is due at 3:52 pm. When is the bus after that due?

(A) 4:23 pm (B) 4:27 pm (C) 4:33 pm (D) 4:30 pm (E) 4:37 pm

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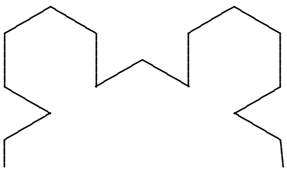
9. A shape is formed when a regular hexagon of side 9 cm has six regular hexagons of side 3 cm added to the outside of it with one at the centre of each side (two of the sides are shown).



What is the perimeter, in centimetres, of the shape?

(A) 72 (B) 126 (C) 144 (D) 162 (E) 180

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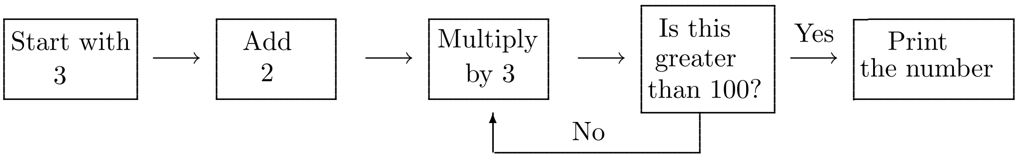
10. Follow the instructions in the flow chart.



What number is printed?

(A) 135 (B) 147 (C) 105 (D) 150 (E) 159

Questions 11 to 20, 4 marks each

11. On my side of the street the houses are numbered 2, 4, 6, 8, 10, 12, 14 and 16. My house is positioned so that the sum of all the house numbers to the left of me is the same as the sum of all those to the right of me. What is my house number?

(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

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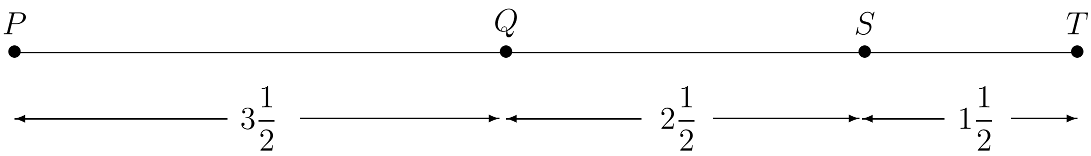
12. The point X (not shown) is the midpoint of QS and the point Y (not shown) is the midpoint of PT.



The length of XY is

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $2\frac{1}{2}$ (E) $3\frac{1}{4}$

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13. The manager of an electrical store bought a brand of TV for \$900. He marked up the price by 50%. However, the TV did not sell so the manager decided to reduce the marked price by 20%. At the new price the TV sold and the result for the store was

(A) \$180 profit (B) \$180 loss (C) \$100 loss (D) no profit or loss (E) \$270 profit

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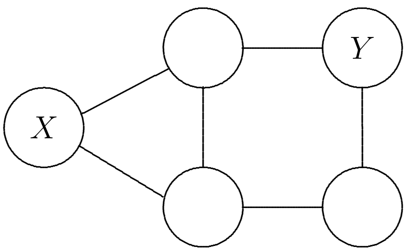
14. Place the numbers 1, 2, 3, 4 and 5, one in each circle in the diagram so that no number is joined by a line to a consecutive number.



The sum of the numbers X and Y could be

(A) 3 (B) 4 (C) 6 (D) 7 (E) 8

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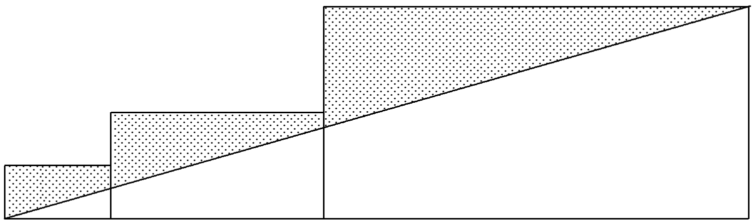
15. Three rectangles are lined up horizontally as shown. The lengths of the rectangles are 2 cm, 4 cm and 8 cm respectively. The heights are 1 cm, 2 cm and 4 cm respectively. A straight line is drawn from the top right-hand corner of the largest rectangle to the bottom left-hand corner of the smallest rectangle.



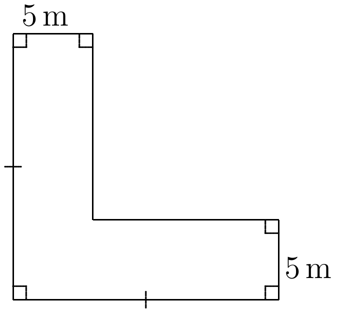
What is the area, in square centimetres, of the shaded region?

(A) 10 (B) 12 (C) 14 (D) 18 (E) 21

16. Anne says 'Bob did it'. Bob says 'Anne is lying'. Chris says 'I did not do it'. Derek says 'Anne did it'. Only one statement is false. The one who did it is

(A) Anne (B) Bob (C) Chris (D) Derek (E) impossible to determine

17. An L-shaped path is 5 m wide and has an area of 125 m^2.



The perimeter, in metres, of the figure is

(A) 35 (B) 40 (C) 45 (D) 60 (E) 75

18. The cells of a 20×20 grid are labelled with the numbers 1, 2, 3, ..., 20 in the first row, 21, 22, 23, ..., 40 in the next row and so on. Which of the numbers below is in one of the four cells touching the centre of the grid at one of its corners?

(A) 189 (B) 199 (C) 200 (D) 211 (E) 220

19. How many four-digit numbers $\boxed{6}\boxed{}\boxed{4}\boxed{}$ are divisible by 36?

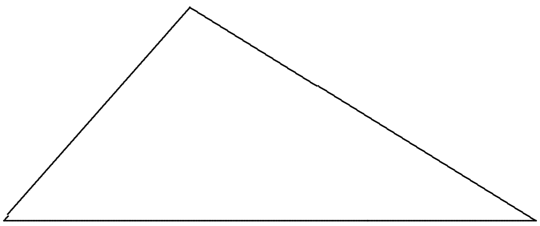
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

20. A number is square-free if the only square number dividing it is 1. For example, 6 is square-free but 12 is not.
How many square-free numbers are there between 90 and 100 inclusive?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Questions 21 to 25, 5 marks each

21. The length of each side of a triangle like the one below is a different prime number and its perimeter is also a prime number.



What is the smallest possible perimeter of such a triangle?

(A) 11 (B) 17 (C) 19 (D) 23 (E) 29

22. Consider the sentence:

THIS IS ONE GREAT CHALLENGE IN MATHEMATICS

Every minute, the first letter of each word is moved to the other end of the word. After how many minutes will the original sentence first reappear?

(A) 422 (B) 880 (C) 1264 (D) 1800 (E) 1980

23. A number a has an equal number of even and odd factors. A number b has an odd number of factors. The sum $a + b$ could be

(A) 14 (B) 16 (C) 17 (D) 20 (E) 21

24. Two mad tilers Arch and Bill are tiling the large foyer of a new building with square tiles. Arch lays the first tile, Bill doubles the area tiled by laying another tile to make a rectangle. Then Arch lays two more tiles to make a square-shaped set of tiles. They keep doubling the area tiled using either a square array of tiles (Arch) or a rectangular array (Bill). At lunchtime they looked at what they had done. Which one of the following statements could be true?

(A) Bill laid the last tile and there are 256 tiles laid.
(B) Arch laid the last tile and there are 2048 tiles laid.
(C) Bill laid the last tile and the overall shape of the tiles is a square.
(D) Bill will lay the next tile after lunch and there are 8192 tiles laid.
(E) Arch will lay the next tile after lunch and there are 512 laid.

25. Eric and Marina each wrote two or three poems every day. Over a period of time, Eric wrote 43 poems while Marina wrote 61. How many days were in this period of time?

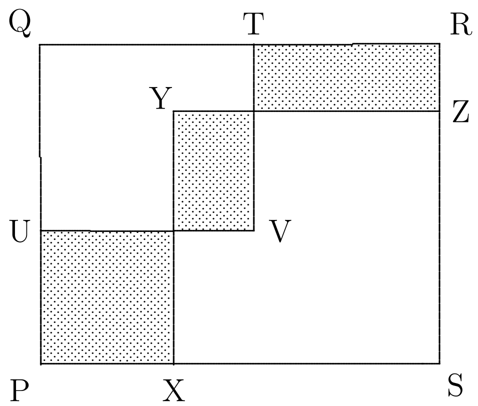
(A) 22 (B) 18 (C) 19 (D) 20 (E) 21

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. An *ascending* number is one in which each successive digit is greater than the one before. A *descending* number is one in which each digit is less than the one before.
Find the 3-digit descending number which is the square of an ascending number.

27. Two overlapping squares, $QTVU$ and $SXYZ$, are drawn inside the rectangle $PQRS$ so that the perimeters of the three shaded rectangles are equal.



If the lengths of the sides of $PQRS$ are 20 cm and 22 cm, what is the sum of the perimeters, in centimetres, of the squares $QTVU$ and $SXYZ$?

28. Consider the three sequences which continue to go up in equal steps:

$$\begin{array}{cccccc} 4, & 9, & 14, & 19, & 24, & \ldots \\ 10, & 21, & 32, & 43, & 54, & \ldots \\ 16, & 33, & 50, & 67, & 84, & \ldots \end{array}$$

What is the first number which occurs in all three sequences?

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29. A 3-digit number is subtracted from a 4-digit number and the *result* is a 3-digit number.

$$\square\square\square\square - \square\square\square = \square\square\square$$

The 10 digits are all different.
What is the smallest possible result?

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30. I have a list of twelve numbers where the first number is 1, the last number is 12 and each of the other numbers is one more than the average of its two neighbours. What is the largest number in the list?

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