AUSTRALIAN MATHEMATICS COMPETITION

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

THURSDAY 4 AUGUST 2011

SENIOR DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 11 AND 12
TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Senior Division

Questions 1 to 10, 3 marks each

1. The expression $3x(x-4)-2(5-3x)$ equals

(A) $3x^2-3x-14$ (B) $3x^2-6x-10$ (C) $3x^2-18x+10$ (D) $3x^2-18x-10$ (E) $9x^2-22x$

2. A coach notices that 2 out of 5 players in his club are studying at university. If there are 12 university students in his club, how many players are there in total?

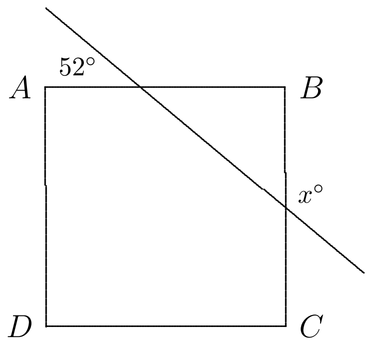
(A) 20 (B) 24 (C) 30 (D) 36 (E) 60

3. The value of $14 \div 0.4$ is

(A) 3.5 (B) 35 (C) 5.6 (D) 350 (E) 0.14

4. In the diagram, $ABCD$ is a square. What is the value of x?

(A) 142 (B) 128 (C) 48 (D) 104 (E) 52



5. Which of the following is the largest?

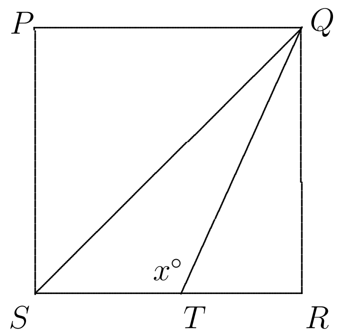
(A) 210 (B) 2^{10} (C) 10^2 (D) 20^1 (E) 21^0

6. If m and n are positive whole numbers and $mn=100$, then $m+n$ cannot be equal to

(A) 25 (B) 29 (C) 50 (D) 52 (E) 101

7. $PQRS$ is a square. T is a point on RS such that $QT = 2RT$.
The value of x is

(A) 100 (B) 110 (C) 120 (D) 150 (E) 160



8. In my neighbourhood, 90% of the properties are houses and 10% are shops. Today, 10% of the houses are for sale and 30% of the shops are for sale. What percentage of the properties for sale are houses?

(A) 9% (B) 80% (C) $33\frac{1}{3}\%$ (D) 75% (E) 25%

9. The value of $\dfrac{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}{2+4+8}$ is

(A) 16 (B) 4 (C) 1 (D) $\frac{1}{4}$ (E) $\frac{1}{16}$

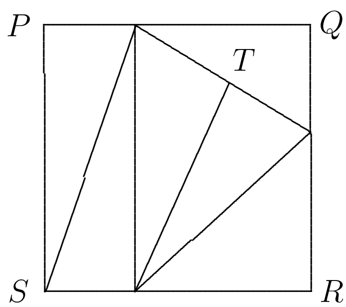
10. Anne's morning exercise consists of walking a distance of 1 km at a rate of 5 km/h, jogging a distance of 3 km at 10 km/h and fast walking for a distance of 2 km at 6 km/h.
How long does it take her to complete her morning exercise?

(A) 30 min (B) 35 min (C) 40 min (D) 45 min (E) 50 min

Questions 11 to 20, 4 marks each

11. The diagram shows a square of side length 12 units divided into six triangles of equal area.
What is the distance, in units, of T from the side PQ?

(A) 4 (B) 3 (C) 2 (D) 1 (E) $\sqrt{5}$



12. Each of the first six prime numbers is written on a separate card. The cards are shuffled and two cards are selected. The probability that the sum of the numbers selected is prime is

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{6}$

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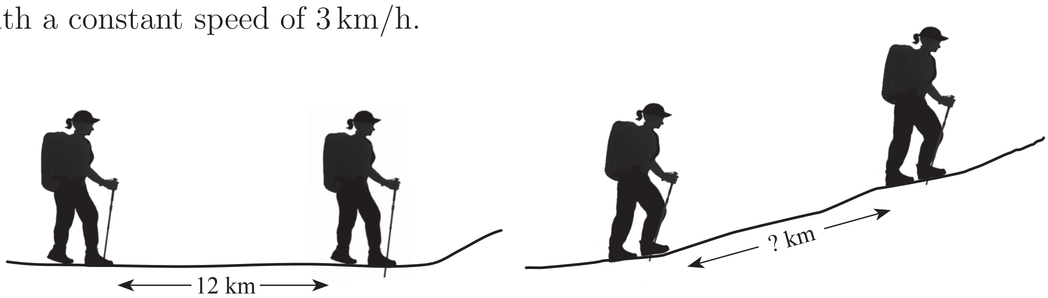
13. Two tourists are walking 12 km apart along a flat track at a constant speed of 4 km/h. When each tourist reaches the slope of a mountain, she begins to climb with a constant speed of 3 km/h.



What is the distance, in kilometres, between the two tourists during the climb?

(A) 16 (B) 12 (C) 10 (D) 9 (E) 8

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14. Lines parallel to the sides of a rectangle 56 cm by 98 cm and joining its opposite edges are drawn so that they cut this rectangle into squares. The smallest number of such lines is

(A) 3 (B) 9 (C) 11 (D) 20 (E) 75

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15. What is the sum of the digits of the positive integer n for which $n^2 + 2011$ is the square of an integer?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

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16. Of the staff in an office, 15 rode a pushbike to work on Monday, 12 rode on Tuesday and 9 rode on Wednesday.
If 22 staff rode a pushbike to work at least once during these three days, what is the maximum number of staff who could have ridden a pushbike to work on all three days?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

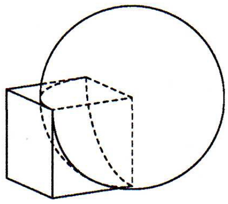
17. How many integer values of n make $n^2 - 6n + 8$ a positive prime number?

(A) 1 (B) 2 (C) 3 (D) 4 (E) an infinite number

18. If $x^2 - 9x + 5 = 0$, then $x^4 - 18x^3 + 81x^2 + 42$ equals

(A) 5 (B) 25 (C) 42 (D) 67 (E) 81

19. The centre of a sphere of radius 1 is one of the vertices of a cube of side 1.



What is the volume of the combined solid?

(A) $\frac{7\pi}{6} + 1$ (B) $\frac{7\pi}{6} + \frac{5}{6}$ (C) $\frac{7\pi}{6} + \frac{4}{3}$ (D) $\frac{7\pi}{8} + 1$ (E) $\pi + 1$

20. In a best of five sets tennis match (where the first player to win three sets wins the match), Chris has a probability of $\frac{2}{3}$ of winning each set. What is the probability of him winning this particular match?

(A) $\frac{2}{3}$ (B) $\frac{190}{243}$ (C) $\frac{8}{9}$ (D) $\frac{19}{27}$ (E) $\frac{64}{81}$

Questions 21 to 25, 5 marks each

21. How many 3-digit numbers can be written as the sum of three (not necessarily different) 2-digit numbers?

(A) 194 (B) 198 (C) 204 (D) 287 (E) 296

22. A rectangular sheet of paper is folded along a single line so that one corner lies on top of another. In the resulting figure, 60% of the area is two sheets thick and 40% is one sheet thick. What is the ratio of the length of the longer side of the rectangle to the length of the shorter side?

(A) $3 : 2$ (B) $5 : 3$ (C) $\sqrt{2} : 1$ (D) $2 : 1$ (E) $\sqrt{3} : 2$

23. An irrational spider lives at one corner of a closed box which is a cube of edge 1 metre. The spider is not prepared to travel more than $\sqrt{2}$ metres from its home (measured by the shortest route across the surface of the box). Which of the following is closest to the proportion (measured as a percentage) of the surface of the box that the spider never visits?

(A) 20% (B) 25% (C) 30% (D) 35% (E) 50%

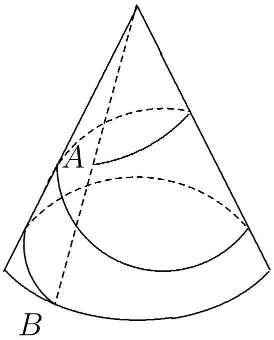
24. Functions f, g and h are defined by

$$\begin{aligned} f(x) &= x+2 \\ g(0) &= f(1) \\ g(x) &= f(g(x-1)) \qquad \text{for } x \geq 1 \\ h(0) &= g(1) \\ h(x) &= g(h(x-1)) \qquad \text{for } x \geq 1. \end{aligned}$$

Find $h(4)$.

(A) 61 (B) 117 (C) 123 (D) 125 (E) 313

25. A cone has base diameter 1 unit and slant height 3 units. From a point A halfway up the side of the cone, a string is passed twice around it to come to a point B on the circumference of the base, directly below A. The string is then pulled until taut.



How far is it from A to B along this taut string?

(A) $\frac{3}{8}(\sqrt{29}+\sqrt{53})$ (B) $\frac{3\sqrt{7}}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{9}{4}$ (E) $\frac{3\sqrt{108}}{8}$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

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26. Paul is one year older than his wife and they have two children whose ages are also one year apart. Paul notices that on his birthday in 2011, the product of his age and his wife's age plus the sum of his children's ages is 2011.
What would have been the result if he had done this calculation thirteen years before?

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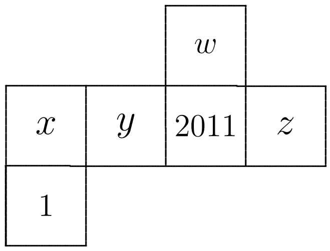
27. The diagram shows the net of a cube. On each face there is an integer: 1, w, 2011, x, y and z.



If each of the numbers w, x, y and z equals the average of the numbers written on the four faces of the cube adjacent to it, find the value of x.

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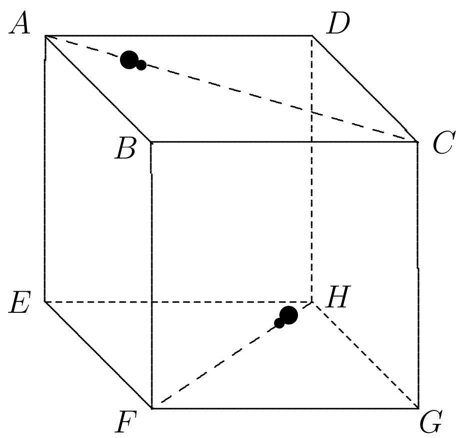
28. Two beetles sit at the vertices A and H of a cube $ABCDEFGH$ with edge length $40\sqrt{110}$ units. The beetles start moving simultaneously along AC and HF with the speed of the first beetle twice that of the other one.



What will be the shortest distance between the beetles?

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29. A family of six has six Christmas crackers to pull. Each person will pull two crackers, each with a different person. In how many different ways can this be done?

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30. A 40×40 white square is divided into 1×1 squares by lines parallel to its sides. Some of these 1×1 squares are coloured red so that each of the 1×1 squares, regardless of whether it is coloured red or not, shares a side with at most one red square (not counting itself). What is the largest possible number of red squares?

A SELECTION OF AUSTRALIAN MATHEMATICS TRUST PUBLICATIONS

Indicate Quantity Required in Box

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These books reproduce the problems and full solutions from both Junior (Years 7 and 8) and Intermediate (Years 9 and 10) versions of the Mathematics Challenge for Young Australians, Challenge Stage. They are valuable resource books for the classroom and the talented student.

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