Australian Mathematics Competition

AN ACTIVITY OF THE AUSTRALIAN MATHEMATICS TRUST

THURSDAY 4 AUGUST 2011

INTERMEDIATE DIVISION COMPETITION PAPER

AUSTRALIAN SCHOOL YEARS 9 AND 10

TIME ALLOWED: 75 MINUTES

INSTRUCTIONS AND INFORMATION

GENERAL

1. Do not open the booklet until told to do so by your teacher.
2. NO calculators, slide rules, log tables, maths stencils, mobile phones or other calculating aids are permitted. Scribbling paper, graph paper, ruler and compasses are permitted, but are not essential.
3. Diagrams are NOT drawn to scale. They are intended only as aids.
4. There are 25 multiple-choice questions, each with 5 possible answers given and 5 questions that require a whole number answer between 0 and 999. The questions generally get harder as you work through the paper. There is no penalty for an incorrect response.
5. This is a competition not a test; do not expect to answer all questions. You are only competing against your own year in your own State or Region so different years doing the same paper are not compared.
6. Read the instructions on the answer sheet carefully. Ensure your name, school name and school year are entered. It is your responsibility to correctly code your answer sheet.
7. When your teacher gives the signal, begin working on the problems.

THE ANSWER SHEET

1. Use only lead pencil.
2. Record your answers on the reverse of the answer sheet (not on the question paper) by FULLY colouring the circle matching your answer.
3. Your answer sheet will be scanned. The optical scanner will attempt to read all markings even if they are in the wrong places, so please be careful not to doodle or write anything extra on the answer sheet. If you want to change an answer or remove any marks, use a plastic eraser and be sure to remove all marks and smudges.

INTEGRITY OF THE COMPETITION

The AMT reserves the right to re-examine students before deciding whether to grant official status to their score.

Intermediate Division

Questions 1 to 10, 3 marks each

1. The value of $2011 - 1102$ is

(A) 1111 (B) 1191 (C) 1001 (D) 989 (E) 909

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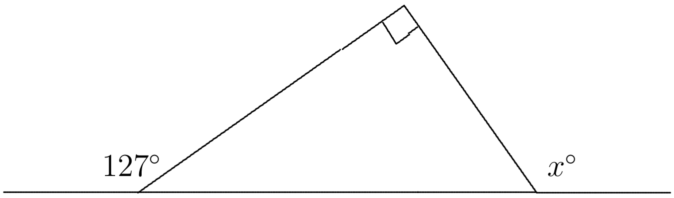
2. In the diagram, the value of x is



(A) 143 (B) 127 (C) 90 (D) 153 (E) 37

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3. The value of $14 \div 0.4$ is

(A) 3.5 (B) 35 (C) 5.6 (D) 350 (E) 0.14

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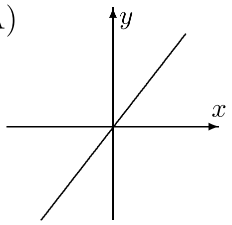
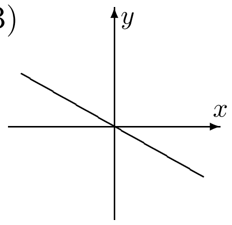
4. Which of the following could be the graph of $y = 2x + 1$?

(A)


(B)


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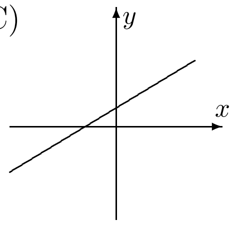
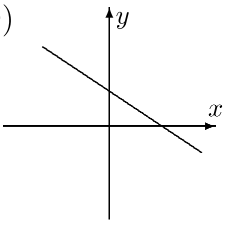
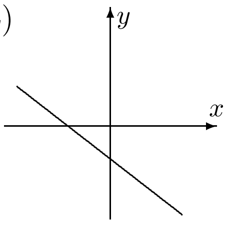


(D)


(E)


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5. The expression $8x - 4y - 3x + 2y$ equals

(A) $4x - y$ (B) $5x - 2y$ (C) $5x - 6y$ (D) $11x - 2y$ (E) $11x - 6y$

6. By what number must $\frac{1}{3}$ be divided to obtain 4 as a result?

(A) $\frac{1}{12}$ (B) 6 (C) $1\frac{1}{3}$ (D) $\frac{1}{4}$ (E) 12

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7. Which one of the following is *not* equal to 3^9?

(A) $(3^3)^3$ (B) $3^3 \times 3^3 \times 3^3$ (C) 27^3 (D) $9^3 \times 27$ (E) 9^4

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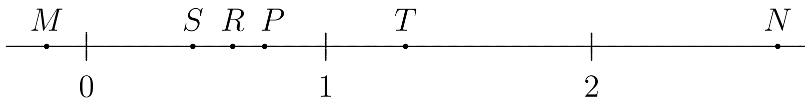
8. The numbers represented by points R and P on the number line below are multiplied. Which point would best represent the product of these two numbers?



(A) M (B) N (C) P (D) S (E) T

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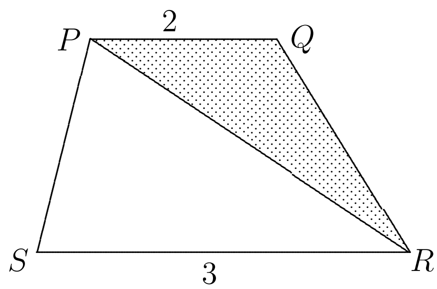
9. $PQRS$ is a trapezium in which $PQ = 2$ units and $RS = 3$ units. What fraction of the trapezium is shaded?



(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{2}{5}$ (E) $\frac{1}{2}$

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10. An $8 \times 8 \times 8$ hollow cube is constructed from $1 \times 1 \times 1$ cubes so that its six walls are 1 cube thick. The number of $1 \times 1 \times 1$ cubes needed to make the hollow cube is

(A) 169 (B) 296 (C) 298 (D) 384 (E) 512

Questions 11 to 20, 4 marks each

11. In my neighbourhood, 90% of the properties are houses and 10% are shops. 10% of the houses are for sale and 30% of the shops are for sale. What percentage of the properties for sale are houses?

(A) 9% (B) 80% (C) $33\frac{1}{3}$% (D) 75% (E) 25%

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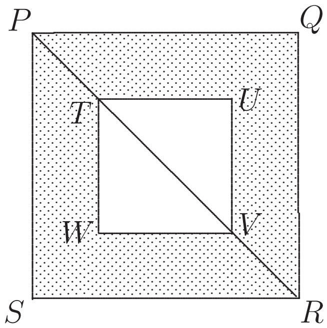
12. $PQRS$ is a square. $TUVW$ is a smaller square placed inside as shown with $PR = 2TV$.
The ratio of the shaded area to the area of the square $PQRS$ is



(A) 2 : 3 (B) 3 : 4 (C) 1 : 3 (D) 1 : 2 (E) 2 : 5

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13. The numbers on the six faces of this cube are consecutive even numbers.

If the sums of the numbers on each of the three pairs of opposite faces are equal, find the sum of all six numbers on this cube.

(A) 196 (B) 188 (C) 210 (D) 186 (E) 198

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14. The positive integers are arranged in a zigzag fashion across five rows as follows:

A	1								9								17				
B		2						8		10						16		18			
C			3				7				11				15				19		
D				4		6						12		14						.	
E					5								13								.

In which row will 2011 appear?

(A) A (B) B (C) C (D) D (E) E

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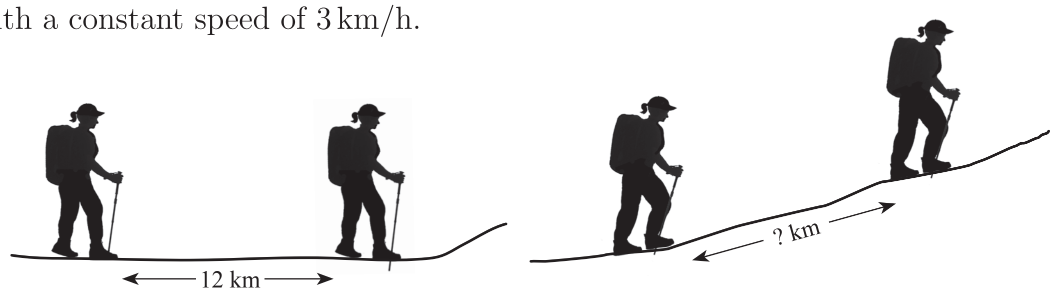
15. Two tourists are walking 12 km apart along a flat track at a constant speed of 4 km/h. When each tourist reaches the slope of a mountain, she begins to climb with a constant speed of 3 km/h.



What is the distance, in kilometres, between the two tourists during the climb?

(A) 16 (B) 12 (C) 10 (D) 9 (E) 8

16. The six faces of a dice are numbered $-3, -2, -1, 0, 1, 2$. If the dice is rolled twice and the two numbers are multiplied together, what is the probability that the result is negative?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{11}{36}$ (D) $\frac{13}{36}$ (E) $\frac{1}{3}$

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17. A 36 cm by 24 cm rectangle is drawn on 1 cm grid paper such that the 36 cm side contains 37 grid points and the 24 cm side contains 25 grid points. A diagonal of the rectangle is drawn. How many grid points lie on that diagonal?

(A) 10 (B) 12 (C) 13 (D) 15 (E) 21

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18. Three people play a game with a total of 24 counters where the result is always that one person loses and two people win. The loser must then double the number of counters that each of the other players has at that time.
At the end of three games, each player has lost one game and each person has 8 counters. At the beginning, Holly had more counters than either of the others. How many did she have at the start?

(A) 9 (B) 11 (C) 13 (D) 16 (E) 24

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19. Mary has 62 square blue tiles and a number of square red tiles. All tiles are the same size. She makes a rectangle with red tiles inside and blue tiles on the perimeter. What is the largest number of red tiles she could have used?

(A) 62 (B) 182 (C) 210 (D) 224 (E) 240

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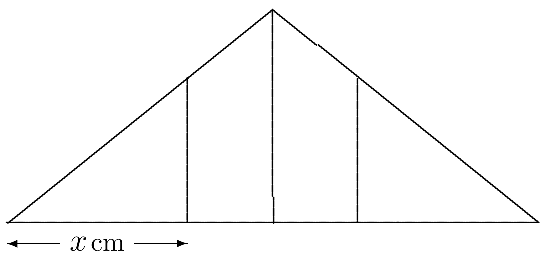
20. An isosceles triangle has a horizontal base of length 12 centimetres. It is divided into four equal areas by three parallel lines as shown.



What is the value of x?

(A) $3\sqrt{2}$ (B) 4 (C) 4.5 (D) 3 (E) $3\sqrt{3}$

Questions 21 to 25, 5 marks each

21. Of the staff in an office, 15 rode a pushbike to work on Monday, 12 rode on Tuesday and 9 rode on Wednesday.
If 22 staff rode a pushbike to work at least once during these three days, what is the maximum number of staff who could have ridden a pushbike to work on all three days?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

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22. I drive a distance of 200 km around the city and my car's average speed is 25 km/h. How far do I then need to drive at an average speed of 100 km/h to raise my car's average speed for the whole time to 40 km/h?

(A) 400 km (B) 200 km (C) 150 km (D) 120 km (E) 100 km

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23. How many 3-digit numbers can be written as the sum of three (not necessarily different) 2-digit numbers?

(A) 194 (B) 198 (C) 204 (D) 287 (E) 296

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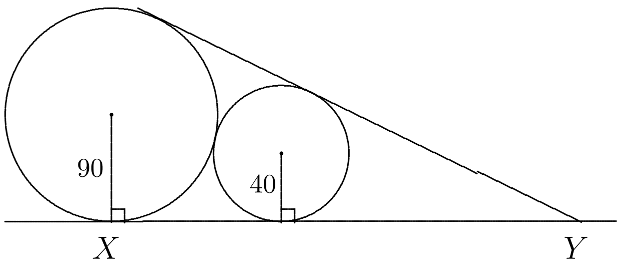
24. A circle of radius 90 units and a circle of radius 40 units are tangent to each other and tangent to two lines as shown in the diagram below.



What is the distance XY?

(A) 120 (B) 180 (C) 216 (D) 234 (E) 260

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25. An arrangement of numbers has *different differences* when the differences between neighbours are all different. For example, the numbers

1	4	2	3

have differences 3, 2 and 1 – all different.

If the numbers from 1 to 6 are arranged with different differences, and with 3 in the third position,

		3			

what is the sum of the last three digits?

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

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26. The first digit of a six-digit number is 1. This digit 1 is now moved from the first digit position to the end, so it becomes the last digit. The new six-digit number is now 3 times larger than the original number. What are the last three digits of the original number?

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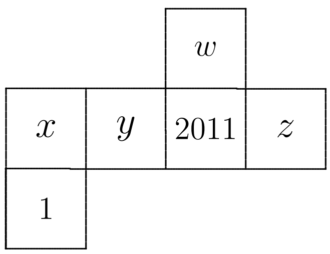
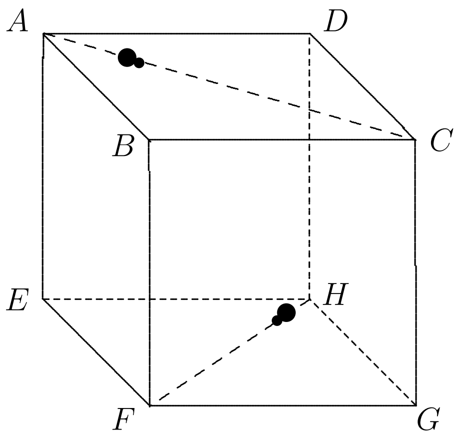
27. The diagram shows the net of a cube. On each face there is an integer: 1, w, 2011, x, y and z.



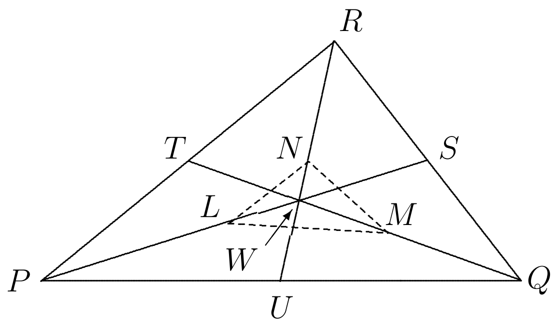
If each of the numbers w, x, y and z equals the average of the numbers written on the four faces of the cube adjacent to it, find the value of x.

28. Two beetles sit at the vertices A and H of a cube $ABCDEFGH$ with edge length $40\sqrt{110}$ units. The beetles start moving simultaneously along AC and HF with the speed of the first beetle twice that of the other one.



What will be the shortest distance between the beetles?

29. In the diagram, $\triangle PQR$ has an area of 960 square units. The points S, T and U are the midpoints of the sides QR, RP and PQ, respectively, and the lines PS, QT and RU intersect at W.



The points L, M and N lie on PS, QT and RU, respectively, such that $PL : LS = 1 : 1$, $QM : MT = 1 : 2$ and $RN : NU = 5 : 4$.
What is the area, in square units, of $\triangle LMN$?

30. A 40×40 white square is divided into 1×1 squares by lines parallel to its sides. Some of these 1×1 squares are coloured red so that each of the 1×1 squares, regardless of whether it is coloured red or not, shares a side with at most one red square (not counting itself). What is the largest possible number of red squares?

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