Senior Division

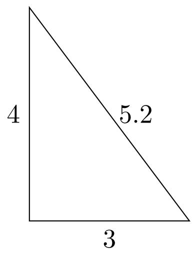
Questions 1 to 10, 3 marks each

1. The expression $\dfrac{0.6}{12}$ is equal to

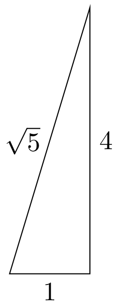
(A) 5 (B) 0.5 (C) 0.05 (D) 0.005 (E) 0.0005

2. Which of the following is a right-angled triangle?

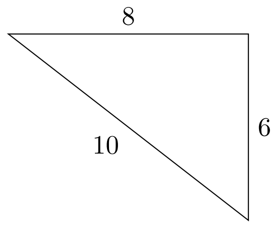
(A)



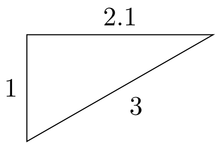
(B)



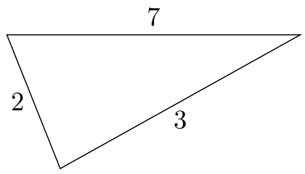
(C)



(D)



(E)



3. The yield from a farm increased by 20% this season to 114 tonnes. Last season the number of tonnes yielded was

(A) 90 (B) 91 (C) 93 (D) 94 (E) 95

4. The cube root of 201 320 132 013 is closest to

(A) 600 (B) 5000 (C) 6000 (D) 50 000 (E) 60 000

5. If $p = 4b + 26$ and b is a positive integer, then p could not be divisible by

(A) 2 (B) 4 (C) 5 (D) 6 (E) 7

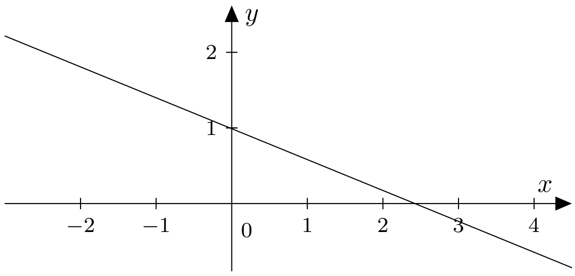
6. The first five terms of a sequence are shown.

$$1, 2, 1, -1, -2, \ldots$$

After the second term, each term can be obtained by subtracting from the previous term the term before that. What is the sum of the first 42 terms of this sequence?

(A) 0 (B) 4 (C) 12 (D) 24 (E) 30

7. A line on a number plane passes through the vertical axis at 1 and the horizontal axis between 2 and 3, as shown. Which of the following equations could be the equation of this line?



(A) $5x + 12y - 12 = 0$ (B) $4x + y + 1 = 0$ (C) $3x - 7y + 7 = 0$ (D) $3x - 2y + 2 = 0$ (E) $2x + 3y - 3 = 0$

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8. When Cody races Chris over 100 metres, Cody wins by 5 metres (that is, when Cody finishes, Chris still has 5 metres to run). When Chris races Joseph over 100 metres, Chris wins by 10 metres. If Cody races Joseph over 100 metres, what will Cody's winning margin be, in metres?

(A) 15.5 (B) 15 (C) 14.5 (D) 14 (E) 13.5

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9. On a ranch in Peru, there are two types of animals, llamas and alpacas. The ranch also has a number of managers. The ratio of llamas to alpacas is 2 : 3 and the ratio of alpacas to managers is 8 : 1. The ratio of animals to managers is

(A) 16 : 3 (B) 13 : 1 (C) 12 : 1 (D) 40 : 3 (E) 20 : 3

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10. Starting with the number 0 on my calculator, I do a calculation in five steps. At each step, I either add 1 or multiply by 2. What is the smallest number that cannot be the final result?

(A) 11 (B) 10 (C) 9 (D) 8 (E) 7

Questions 11 to 20, 4 marks each

11. A box contains three bags. One bag contains one white pebble and three black pebbles. Another bag also contains one white pebble and three black pebbles. The remaining bag contains one white pebble and four black pebbles. You reach into the box and randomly pull out a bag, then reach into the bag and randomly pull out a pebble. What is the probability that this pebble is white?

(A) $\frac{7}{30}$ (B) $\frac{3}{13}$ (C) $\frac{1}{9}$ (D) $\frac{1}{4}$ (E) $\frac{11}{36}$

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12. A region on a number plane is enclosed by the x-axis, the y-axis, the line $x = 2$ and the graph of the function $y = f(x)$ where

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 1 + \sqrt{1 - (x-2)^2} & \text{for } 1 < x \le 2 \end{cases}$$

The area of this region, in square units, is

(A) $\frac{5}{2}$ (B) $1 + \frac{\pi}{4}$ (C) $1 + \frac{\pi}{2}$ (D) $2 + \pi$ (E) $2 + \frac{\pi}{4}$

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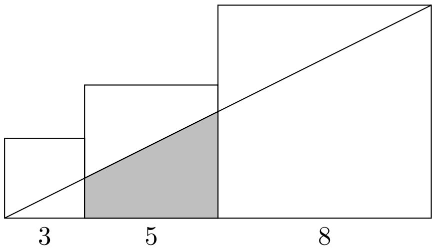
13. Three squares, with side lengths 3 cm, 5 cm and 8 cm, are arranged in a row as shown.



The area, in square centimetres, of the shaded trapezium is

(A) 12 (B) $\frac{73}{6}$ (C) $\frac{55}{4}$ (D) 14 (E) $\frac{25}{2}$

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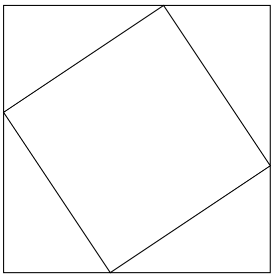
14. A square of perimeter 20 cm is inscribed in a square of perimeter 28 cm. What is the greatest distance between a vertex of the inner square and a vertex of the outer square, in centimetres?



(A) $\sqrt{58}$ (B) $\frac{7\sqrt{5}}{2}$ (C) 8 (D) $\sqrt{65}$ (E) $5\sqrt{3}$

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15. Six different two-digit numbers are formed using three different non-zero digits. The sum of five of those six numbers equals 100. What is the sixth number?

(A) 23 (B) 32 (C) 45 (D) 54 (E) 67

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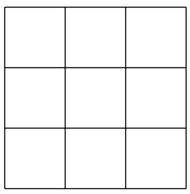
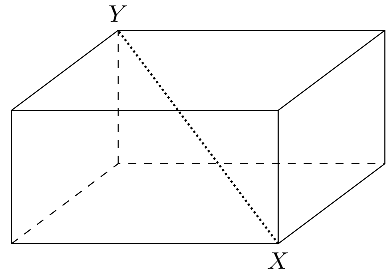
16. Three darts are thrown at the 3×3 grid, each landing in a different small square. After each throw, each of the remaining small squares is equally likely to be hit.



What is the probability that the squares in which they land form a horizontal, vertical or diagonal row?

(A) $\frac{1}{63}$ (B) $\frac{2}{21}$ (C) $\frac{1}{9}$ (D) $\frac{1}{42}$ (E) $\frac{8}{81}$

17. What is the largest possible volume of a box with rectangular faces of integer dimensions where the long diagonal $XY = 9$?



(A) 32 (B) 81 (C) 90 (D) 108 (E) 112

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18. The number of different ordered pairs (x, y) which satisfy the equation

$$2x^2 - 2xy + y^2 = 169$$

where x and y are integers and $x \geq 0$ is

(A) 2 (B) 4 (C) 5 (D) 7 (E) 8

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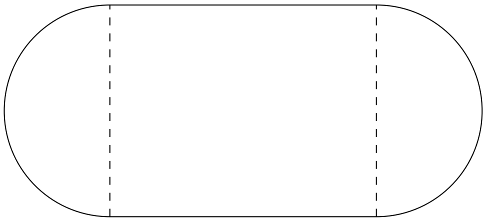
19. A running track encloses an area formed by a rectangle joined to two semicircles, as shown in the diagram below. If the length of the track must be 400 metres, then what is the maximum possible area of the rectangle, in square metres?



(A) $\dfrac{160\,000}{(\pi+2)^2}$ (B) $\dfrac{20\,000}{\pi}$ (C) $\dfrac{30\,000}{\pi}$ (D) $10\,000$ (E) $\dfrac{40\,000}{\pi}$

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20. In how many ways can 2013 be expressed as the sum of two or more consecutive positive integers?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

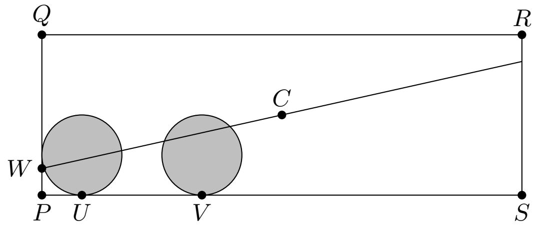
Questions 21 to 25, 5 marks each

21. If $x^2 = x + 3$ then x^5 equals

(A) $7x + 12$ (B) $12x + 7$ (C) $17x + 17$ (D) $19x + 21$ (E) $21x + 19$

22. The rectangle $PQRS$ shown has $PQ = 4$, $PS = 12$ and centre C. The two shaded circles have radius 1 and touch PS at U and V where $PU = 1$ and $PV = 4$.

The line CW divides the unshaded area in half. The length of PW is



(A) $\frac{2}{7}$ (B) $\frac{2}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

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23. Three non-zero real numbers x, y, z can be found such that

$$\sqrt{x+y} + \sqrt{y+z} = \sqrt{z+x}.$$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ must be equal to

(A) 0 (B) 1 (C) -1 (D) xyz (E) $x+y+z$

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24. A two-digit number is written on the blackboard and five students make the following statements about the number.

Amy: The number is prime.
Ben: The number can be expressed as the sum of two perfect squares.
Con: At least one of the digits in the number is a 7.
Dan: The number obtained by reversing the digits is odd and composite.
Eve: The number differs from a prime number by 2.

If only one of the students is wrong, who is it?

(A) Amy (B) Ben (C) Con (D) Dan (E) Eve

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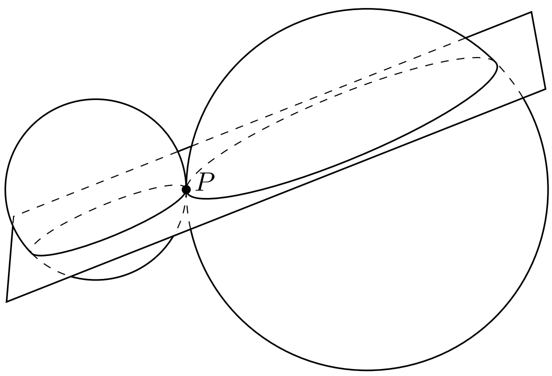
25. Two spheres, one of radius 1 and the other of radius 2, touch externally at P. A plane through P cuts the volume of the shape formed by these two spheres in the ratio $1 : 2$. In what ratio does this plane cut the volume of the smaller sphere?



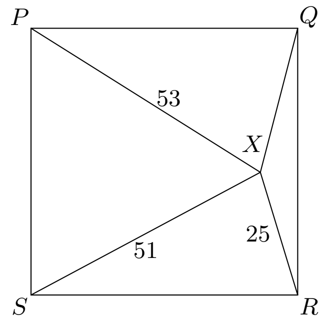
(A) $1 : 2$ (B) $4 : 9$ (C) $1 : 3$ (D) $4 : 11$ (E) $2 : 5$

For questions 26 to 30, shade the answer as an integer from 0 to 999 in the space provided on the answer sheet.

Question 26 is 6 marks, question 27 is 7 marks, question 28 is 8 marks, question 29 is 9 marks and question 30 is 10 marks.

26. A hockey game between two teams is 'relatively close' if the number of goals scored by the two teams never differ by more than two. In how many ways can the first 12 goals of a game be scored if the game is 'relatively close'?

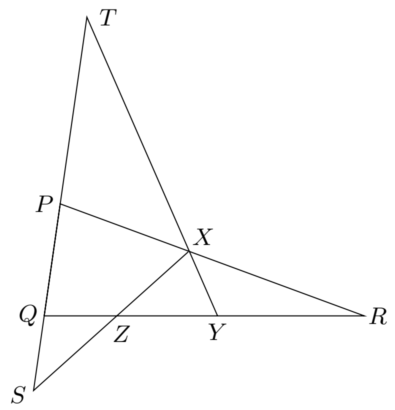
27. The point X is inside the square $PQRS$. X is 25 m from R, 51 m from S and 53 m from P. The distance of X from each side of the square is an integer number of metres. What is the area, in square metres, of $\triangle PQX$?



28. All the digits of the positive integer N are either 0 or 1. The remainder after dividing N by 37 is 18. What is the smallest number of times that the digit 1 can appear in N?

29. The points X, Y and Z are on the sides of $\triangle PQR$ as shown, such that

$$QZ : ZY : YR = 1 : 2 : 3 \text{ and } PX : XR = 4 : 5$$



If $QS = 11$ cm, find the length of ST, in centimetres.

30. An acute-angled triangle lies in the plane such that the coordinates of its vertices are all different integers and no sides are parallel to the coordinate axes. If the triangle has area 348 and one side of length 29, what is the product of the lengths of the other two sides?